Stopping near the top of a random walk

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Abstract

This poster discusses the problem of maximizing the probability of stopping with one of the two highest values in a Bernoulli random walk with arbitrary parameter p and finite time horizon n. The optimal strategy (continue or stop) depends on a sequence of threshold values which has an intriguing oscillating pattern. Several properties of this sequence were proven and others conjectured in a 2010 paper by P. Allaart. This poster will discuss recent progress toward proving the conjectures.

Introduction

Definition. A stopping time with respect to a sequence of random variables $X_1, X_2, \ldots$ is a random variable τ with values in $\{1,2,\ldots\}$ and the property that for each τ in $\{1,2,..\}$, the occurrence or non-occurrence of the event $\{\tau = t\}$ depends only on the values of $X_1, X_2, \ldots X_t$.

Stopping near the top of a random walk

(i) Let $X_1, X_2, \ldots X_N$ be independent Bernoulli random variables with parameter p
(ii) Consider $S_0:=0$, $S_n:= X_1+X_2+\ldots+X_n$ for $n < N$ and
(iii) $M_N:= max(S_0,S_1,\ldots S_N)$

Problem
Given $N>0$, find a stopping time $\tau \le N$ so as to maximize $P(M_N - S\tau \le 1)$.
Win if we stop at one of the two highest values.

Exploration
Say we are in state *(n,i)* if:
There are n steps remaining until the time horizon *N*;
The walk is currently *i* units below its running maximum.

Obviously, it is optimal to continue in states *(n,2), (n,3),…*

Lemma (Allaart)
In state *(n,0)* with $n \geq 1$, it is also optimal to continue.
The critical states are *(n,1)*, for *n=1,2,…*

Lemma (Allaart)
For each $n \geq 1$, there exists $0 < p_n \le 1$ such that, in state *(n,1)*, it is optimal to
stop if $p \le p_n$;
continue if $p \ge p_n$.

Remark
The p_n can be calculated by backward induction.

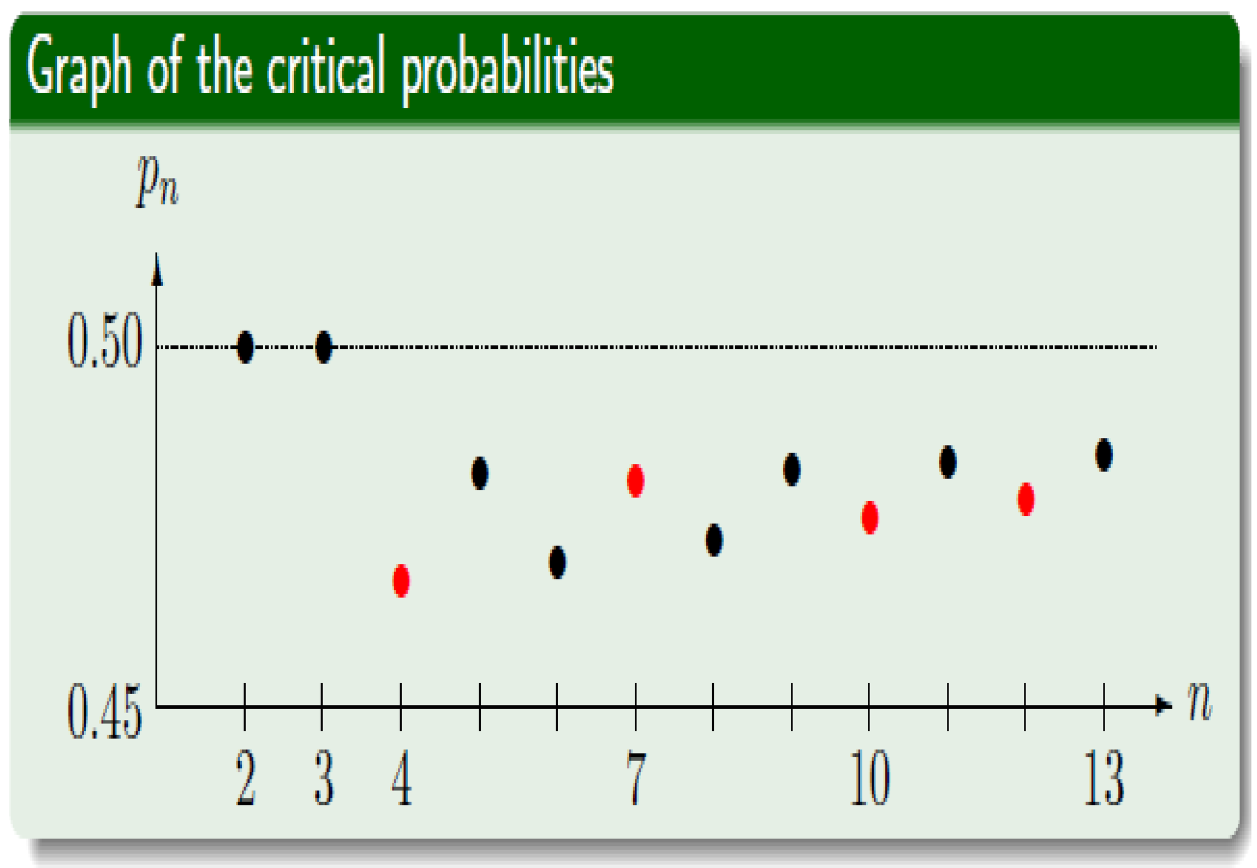
Properties of the sequence

Theorem (Allaart)
(i) $p_n < 0.5$ for all $n \ge 4$
(ii) $\limsup p_n = 0.5$
(iii) $p_{2n} < p_{2n-1} < p_{2n+1}$ for all $n \ge 4$,

Conjectures (Allaart)
(i) $\lim p_n = 0.5$
(ii) $p_{2n} < p_{2n+2}$ for $n \ge 2$

Results

Theorem (J.A.I.)
(i) $p_n \ge p_4$ for all $n \ge 1$
(ii) $p_{2n+4} \le p_{2n+1}$ for $n \ge 0$
(iii) $p_{2n+6} \le p_{2n+1}$ for $n \ge 3$



References

Allaart, P. C. (2010). How to stop near the top in a random walk?. Decision Making Processes under Uncertainty and Ambiguity, RIMS Kokyuroku 1682} 33--40.

Hlynka, M. and Sheahan, J. N. (1988). The secretary problem for a random walk. Stoch. Proc. Appl. 28, 317--325.

Yung, S. P. and Zhou, W. (2009). Two rationales behind `buy-and-hold or sell-at-once'. J. Appl. Probab. 46, 651--668.

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